

7.5

$$\#2 \quad y'' - y' - 2y = 0, \quad y(0) = -2$$

$$y'(0) = 5$$

$$\left( s^2 Y(s) - s y(0) - y'(0) \right) - \left( s Y(s) - y(0) \right) - 2Y(s) = 0$$

$$\underline{s^2 Y(s)} + 2s - 5 - \underline{s Y(s)} - 2 - \underline{2Y(s)} = 0$$

$$(s^2 - s - 2)Y(s) + 2s - 7 = 0$$

$$Y(s) = \frac{-2s + 7}{(s-2)(s+1)}$$

$$\frac{-2s + 7}{(s-2)(s+1)} = \frac{1}{s-2} + \frac{-3}{s+1}$$

$$\therefore y(t) = \mathcal{F}^{-1} \left\{ \frac{1}{s-2} \right\} - 3 \mathcal{F}^{-1} \left\{ \frac{1}{s+1} \right\}$$

$$y(t) = e^{2t} - 3e^{-t}$$

7.8

$$\#6 \quad y'' - 4y' + 5y = 4e^{3t}$$

$$y(0) = 2$$

$$y'(0) = 7$$

$$(s^2 Y(s) - s y(0) - y'(0)) - 4(s Y(s) - y(0)) + 5 Y(s) = \frac{4}{s-3}$$

$$\underline{s^2 Y(s)} - 2s - 7 - 4 \underline{s Y(s)} + 8 + \underline{5 Y(s)} = \frac{4}{s-3}$$

$$(s^2 - 4s + 5) Y(s) = \frac{4}{s-3} + 2s - 1 = \frac{4 + (2s-1)(s-3)}{s-3}$$

$$Y(s) = \frac{4 + (2s-1)(s-3)}{(s-3)((s-2)^2 + 1)}$$

$$\frac{4 + (2s-1)(s-3)}{(s-3)((s-2)^2 + 1)} = \frac{A}{s-3} + \frac{B(s-2)}{(s-2)^2 + 1} + \frac{C}{(s-2)^2 + 1}$$

$$\text{let } s=2 \quad \frac{1}{-1} = -2 + C \Rightarrow C=1$$

$$\text{let } s=0 \quad \frac{7}{-15} = -\frac{2}{3} - \frac{2B}{5} + \frac{1}{5}$$

$$-7 = -10 - 6B + 3$$

$$-7 = -7 - 6B$$

$$0 = B$$

$$\therefore y(t) = \underline{2e^{3t} + e^{2t} \sin t}$$

7.5

$$\#7 \quad y'' - 7y' + 10y = 9\cos t + 7\sin t,$$

$$y(0) = 5$$

$$y'(0) = -4$$

$$\left( s^2 Y(s) - s y(0) - y'(0) \right) - 7 \left( s Y(s) - y(0) \right) + 10 Y(s) = \frac{9s}{s^2+1} + \frac{7}{s^2+1}$$

$$\underline{s^2 Y(s)} - 5s + 4 - 7 \underline{s Y(s)} + 35 + \underline{10 Y(s)} = \frac{9s+7}{s^2+1}$$

$$(s^2 - 7s + 10)Y(s) - 5s + 39 = \frac{9s+7}{s^2+1}$$

$$(s^2 - 7s + 10)Y(s) = \frac{9s+7}{s^2+1} + 5s - 39 = \frac{9s+7 + (5s-39)(s^2+1)}{s^2+1}$$

$$Y(s) = \frac{9s+7 + (5s-39)(s^2+1)}{(s^2+1)(s-2)(s-5)}$$

$$\text{So } \frac{9s+7 + (5s-39)(s^2+1)}{(s^2+1)(s-2)(s-5)} = \frac{8}{s-2} + \frac{-4}{s-5} + \frac{1}{s^2+1} + \frac{0}{s^2+1}$$

$$\text{let } s=0 \quad \frac{-32}{10} = -4 + \frac{4}{5} + D$$

$$-32 = -40 + 8 + D$$

$$-32 = -32 + D$$

$$0 = D$$

$$\text{let } s=1 \quad \frac{-52}{8} = -8 + 1 + \frac{c}{2}$$

$$\frac{-52}{8} = -7 + \frac{c}{2} \Rightarrow -52 = -56 + 4c$$

$$4 = 4c$$

$$c=1$$

7.5

H 7 cont.

$$Y(s) = \frac{8}{s-2} - \frac{4}{s-5} + \frac{s}{s^2+1}$$

$$\therefore y(t) = \underline{8e^{2t} - 4e^{5t} + \cos(t)}$$